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On a Method to Quantify the Far-Field Uncertainty of Array Antennas with Respect to Uncertainties of Antenna Current Densities

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Summary

An important quantity in investigations of antenna arrays is their far-field behavior. Imprecise knowledge of far-field uncertainties of an array is rather detrimental and might cause the overall system to fail. It is therefore essential to have good control of uncertainties and their propagation through the array system. It is well known that the spatial Fourier transform is an essential part to relate the generating current densities to their associated far fields. This map is a linear map, and examinations of the robustness properties of the far field are hence directly connected to the properties of the Fourier transform of the generating currents. In this work, we use stochastic methods to determine the propagation of uncertainties from the antenna current densities to the far field. The work provides the relation between the probability distribution of the far field in each direction to the probability distribution of the spatial variations of the current density. This relation is reduced to a one-dimensional finite integral under the assumption that the current distribution is Gaussian. For the investigated cases a strong robustness is observed.

1 Introduction

The far field of an array is one of the main parameters in its design. Perturbations and deviations of the desired far-field can result in the proposed device's failure to satisfy its purpose. It may not fulfill its specification or possibly not remain within the allowed regulatory range governing its behavior. It is therefore interesting to determine to what extent the far field is robust with respect to perturbations from the generating sources. A common theme of this kind of investigation has been perturbations with respect to the local geometry of the device. Important early work was done in [1, 2]. More recent results include [3, 4], who investigated the dependence of uncertainties in the signal, and the standard deviation with respect to uncertainty in the phase and/or the position of the element of the arrays, respectively. Another recent approach to geometrical uncertainties is applied in [5].

In this work, the focus is on a more numerical perspective of the perturbations in the far field. In most numerical approximations of antennas and scatterers, the far field is determined by a Fourier transform of the field over an enclosing surface or volume of the antenna. In the case of the method of moments, the unknown quantity obtained by the numerical solution is the surface (or volume) current. In the case of FDTD and FEM, it is sufficient to know the fields on an enclosing surface and the spatial Fourier-transform over the surface of a linear combination of the equivalent currents determines the associated far field. The question that is considered in this work is how the uncertainty of the (equivalent) currents are related to the uncertainty of the far-field.

2 Results

A theoretical and high-dimensional integral relation describing how the probability density distribution of the far field is related to the probability distribution of the current density at each spatial point of the array is determined. We make the assumption that the real and imaginary parts of the current density follow a Gaussian probability distribution at each spatial point over the support of the current density. From this assumption, we show that the high-dimensional integral relation between these probability densities can be reduced to an explicit expression containing a one-dimensional finite integral. This integral is straightforward to integrate numerically.

Thus, provided with a numerical representation of the antenna array current, the above theory explicitly determines the probability density of the far field in any desired propagation direction. Derived quantities like the standard deviation can be extracted. In all the investigated cases, high robustness is demonstrated. E.g., rather large perturbations of the current density result in small variations of the resulting far field. This can be quantized for example with a reduction of the standard for the far fields as compared with the normalized standard deviation of the current densities. To illustrate this robustness effect, consider an 8×1

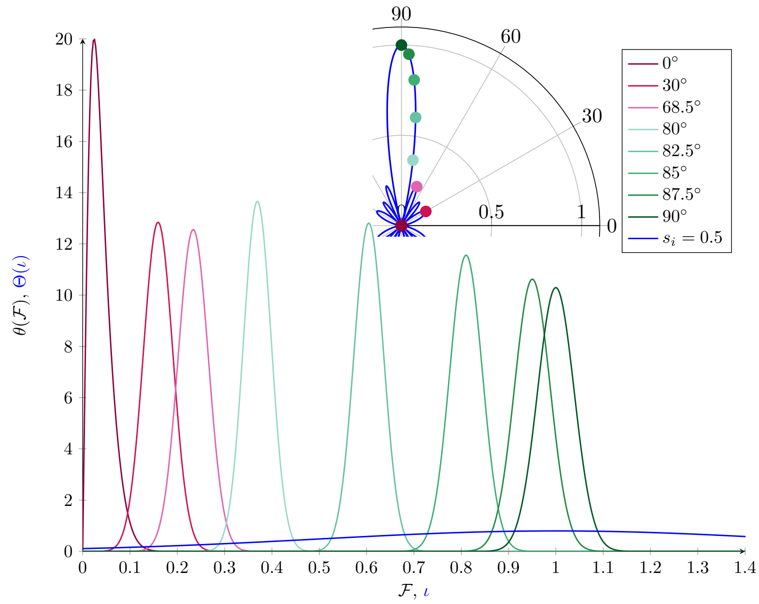

Figure 1. This result is for a linear array of eight bowtie elements in free space. The slowly varying blue line corresponds to the (normalized) probability density function $\Theta(\iota)$ of the current density over the spatial mesh of the array. The eight colored graphs, from red to green, represent the probability density function $\theta(\mathscr{F})$ in directions ranging from 0 to 90°, as detailed in the legend. The directions are also marked on the radiation pattern in the inset.

array of bowtie elements. The unperturbed radiation pattern is shown in the inset in Fig. 1. On the radiation pattern at given angles, there is a range of colored dots, the corresponding color in the legend, and the main graph of Fig. 1 is the probability distribution at that angle of the magnitude of the far field. The Gaussian standard deviation of spatial points of the current density is denoted with s_i and its probability density is depicted in blue in the main graph in Fig. 1. Note that the (normalized) standard deviation of the underlying current here is 0.5, but the resulting standard deviation of the field in any given direction is a factor 10 smaller.

Additional examples and illustrations of the uncertainty propagation will be presented at the Swedish Microwave Days 2023.

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